1 Vector/scalar Baryon and Lepton number conserving leptoquarks

In this chapter, we want to...

- meet some leptoquarks who can live at the TeV
- see that at $p^2 \ll m_{LQ}^2$, they induce 2-quark 2-lepton "contact" interactions at tree level (like the W gives 4-fermion "Fermi" interaction)
- massage 4-f interactions to convenient form using Fierz transformations, etc.
- set contraints from the PDB*lattice

Why should you consider the possibility of leptoquarks?

- why not?
- hadron colliders are great places to find them
- theorists expections for the TeV are not verified so far, and theorists don't like LQ
- the SM contains coloured bosons and charged bosons, why not coloured+charged bosons?
- anomaly cancellation knows that there are leptons and quarks; why are there no leptoquarks?

1.1 Meet the leptoquark zoo

- leptoquark ≡ boson interacting with a lepton or quark
- focus here on leptoquarks which conserve B and L (avoid p decay bounds)
- 7 (respectively, for scalars and vectors) renormalizable B and L conserving quark-lepton-boson interactions, consistent with the SU(3)×SU(2)×U(1) gauge symmetry of the Standard Model. In scalar case:

  Buchmuller,Ruckl, Wyler

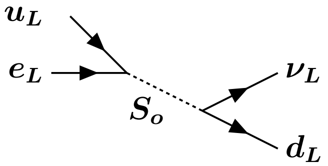
  “Aachen notn”

$$
\begin{aligned}
\mathcal{L}_S \;=\; & -\{\, \boldsymbol{(\lambda_{LS_o}\overline{q^c} i\sigma_2 \ell + \lambda_{RS_o}\overline{u^c} e) S_o^\dagger} + \lambda_{R\tilde{S}_o}\overline{d^c} e \tilde{S}_o^\dagger \\
& +(\lambda_{LS_{1/2}}\overline{u}\ell + \lambda_{RS_{1/2}}\overline{q} i\sigma_2 e) S_{1/2}^\dagger + \lambda_{L\tilde{S}_{1/2}}\overline{d}\ell \tilde{S}_{1/2}^\dagger \\
& +\lambda_{LS_1}\overline{q^c} i\sigma_2 \vec{\sigma}\ell \cdot \vec{S}_1^\dagger \,\} + \boldsymbol{h.c.}
\end{aligned}
\tag{1.15}
$$

- $\ell^T = (\nu_L, e_L)$, $q^T = (u_L, d_L)$ are left-handed $SU(2)_L$ doublets, $e = e_R$, $u = u_R$ and $d = d_R$ are right-handed charged singlets.
- Charge conjugate fields : $\overline{\ell^c} = (\ell^c)^\dagger \gamma^o = -\ell^T C^{-1}$.

  Also, (from P+S defn of Pauli matrices, A.21) $i\sigma_2 = \varepsilon = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
- Each coupling λ carries generation indices for the two fermions (suppressed in equations (1.15)).
- the SU(2) singlet, doublet and triplet leptoquarks respectively have subscipts 0, 1/2 and 1, and the L/R index on the coupling reflects the lepton chirality.
- neglect the vector here because...I don't know how to renormalise massive vectors

1.2 An example!



Focus on first generation fermions, singlet S_o

$$\begin{aligned}\mathcal{L}_S \ni -(\lambda_{LS_o}\overline{q^c} i\sigma_2 \ell + \lambda_{RS_o}\overline{u^c}e)S_o^\dagger + h.c. &= -\left(\lambda_{LS_o}\left[\begin{array}{cc}\overline{(u_L)^c} & \overline{(d_L)^c}\end{array}\right]\left[\begin{array}{cc}0 & 1\\ -1 & 0\end{array}\right]\left[\begin{array}{c}\nu_L\\ e_L\end{array}\right] + \lambda_{RS_o}\overline{u^c}e\right)S_o^\dagger + h.c.\\ &= \left(\lambda_{LS_o}\overline{(u_L)^c}e_L - \lambda_{LS_o}\overline{(d_L)^c}\nu_L + \lambda_{RS_o}\overline{u^c}e_R\right)S_o^\dagger + h.c. \end{aligned} \tag{1.16}$$

makes amplitudes → 4-f contact interactions:

$$\begin{aligned}-i\lambda_{LS_o}(\overline{(u_L)^c}e_L - \overline{(d_L)^c}\nu_L)\frac{i}{p^2-m_S^2}(+i)\lambda^*_{LS_o}(\overline{(u_L)^c}e_L - \overline{(d_L)^c}\nu_L)^\dagger &\rightarrow \boldsymbol{-i\frac{|\lambda_{LS_o}|^2}{m_S^2}}[(\overline{(u_L)^c}e_L)(\overline{(u_L)^c}e_L)^\dagger - (\overline{(u_L)^c}e_L)(\overline{(d_L)^c}\nu_L)^\dagger\\ &\quad \boldsymbol{-(\overline{(d_L)^c}\nu_L)(\overline{(u_L)^c}e_L)^\dagger} + (\overline{(d_L)^c}\nu_L)(\overline{(d_L)^c}\nu_L)^\dagger] \end{aligned} \tag{1.17}$$

$$\begin{aligned}-i\lambda_{RS_o}(\overline{(u_R)^c}e_R)\frac{i}{p^2-m_S^2}(+i)\lambda^*_{RS_o}(\overline{(u_R)^c}e_R)^\dagger &\rightarrow -i\frac{\lambda_{RS_o}\lambda^*_{RS_o}}{m_S^2}(\overline{(u_R)^c}e_R)(\overline{(u_R)^c}e_R)^\dagger\\ -i\lambda_{RS_o}(\overline{(u_R)^c}e_R)\frac{i}{p^2-m_S^2}(+i)\lambda^*_{LS_o}(\overline{(u_L)^c}e_L) - \overline{(d_L)^c}\nu_L)^\dagger + h.c. &\rightarrow \boldsymbol{i\frac{\lambda_{RS_o}\lambda^*_{LS_o}}{m_S^2}(\overline{(u_R)^c}e_R)(\overline{(d_L)^c}\nu_L)^\dagger} + \ldots \end{aligned} \tag{1.18}$$

Lets focus on the bold face ones which have which have the quantum numbers to mediate $\pi^- \rightarrow e\bar{\nu}$. But the SM amplitude for $\pi \rightarrow e\bar{\nu}$ via W exchange has the form (F rule for $W\overline{u}\gamma P_L d$ is $ig/\sqrt{2}$):

$$-\frac{g^2}{2m_W^2}(\overline{u_L}\gamma^\mu d_L)(\overline{e_L}\gamma_\mu\nu_L) \tag{1.19}$$

To constrain the LQ contribution, it would be convenient to re-express (1.17,1.18) more similar to the SM amplitude... possible via Fierz Transformations!

1.3 Signs, is and 2s in contact interactions

To construct a contact interaction, which resides in the Lagrangian, from an amplitude computed with Feynman rules, one must first take the limit of the amplitude such that it becomes a "contact amplitude", then identify the Lagrangian interaction which could induce that amplitude.

1.3.1 To obtain the heavy-particle-mediated amplitude

Ideally, one has Feynman rules for the heavy particle. However, if one only has a Lagrangian, F-rules can be guessed up to combinatoric factors.

- Suppose the Lagrangian of a heavy particle ($\equiv \phi$) contains interactions with fermions of the form

$$-g\overline{Q}\Gamma L\phi \in \mathcal{L} \tag{1.20}$$

  where $g \in C$ a coupling constant, Q, L are *chiral* fermions, $\Gamma = \gamma^{\mu}$, or 1, and ϕ a vector (in which case contracted with Γ) or a scalar. The $-$ve sign is because $\mathcal{L}$ = kinetic - potential energy.

- Factors of i in F-rules: (not so sure here?) F-rule for the vertex is a time-ordered expectation value between the desired initial and final states. In the Minkowski path integral $\exp\{i \int \mathcal{L}\}$ (see Peskin+Schroeder 9.30), evaluating perturbatively, the interaction Lagrangian comes down from the exponent to be written as a power series in $i\mathcal{L}$. So the Feynman rule for (1.20) will be $-ig$, and $-ig^*$ for the *h.c.* vertex. Important point is that i has same sign for vertex and its *hc*.

- factors of 2 and combinatorics (even less sure here): if a field appears twice in an interaction (eg, majorana fermion or real scalar field mass terms), then there are two ways of propagating the asymptotic states to the interaction, and you get a 2 in the F-rule. Alternatively, imagine that the F-rule is derivative of the Lagrangian wrt the field.

1.3.2 The local limit of the amplitude

- to obtain the amplitude for a 4-fermion interaction, mediated at tree-level by a boson whose mass exceeds the energy scale of the experiment, one neglects the momentum in the propagator. The scalar and vector propagators therefore become (Note the relative sign!)

  $$\frac{i}{p^2 - m^2} \to \frac{-i}{{m_L}^2} \qquad , \qquad \frac{-ig^{\mu\nu}}{p^2 - m^2} \to \frac{+ig^{\mu\nu}}{{m_L}^2}$$

  For example, this gives local four fermion amplitudes mediated by a scalar or vector as

  $$\frac{i|\lambda|^2}{m_L^2}(\overline{Q}_1 P_X L_1)(\overline{L}_2 P_Y Q_2) \qquad , \qquad \frac{-i|\lambda|^2}{m_L^2}(\overline{Q}_1 \gamma^\mu P_X L_1)(\overline{L}_2 \gamma_\mu P_Y Q_2) \tag{1.21}$$

  where Q_1, Q_2, L_1, L_2, are now 4-component Dirac spinors (usually u, v).

1.3.3 A contact interaction in the Lagrangian

It remains to construct a contact term in the Lagrangian.

- remove a factor i from the " contact amplitude".
- Replace the 4-comp spinors by the relevant fields (P+S 3.99 and 3.100 expand ψ on u and v, and $\overline{\psi}$ on $\overline{u}$ and $\overline{v}$).
- 2s...if an identical field appears twice in the amplitude, divide the interaction by 2.
- put the interaction in $\mathcal{L}$, *and add* $+h.c.$! (unless interaction is self-conjugate).

Applying this recipe to (1.21), gives that the Fermi interaction will appear in $\mathcal{L}$ with a negative sign, opposite to the 4fv induced by scalars. This agrees with Cheng and Li.

1.4 Rewriting fermion contractions: Fierz, etc

Appendix A:Haber and Kane
Cheng+Li Appendix

Peskin+Schroeder 3.4

By appropriate Fierz transformations and transpositions of matrix elements, four-fermion vertices mediated by scalars interacting with fermions of only one chirality (ie if write vertex in Dirac notn, always get same chiral projection op when the scalar is created) can all be brought into $(V \pm A)(V \pm A)$ or $(V + A)(V - A)$ form.

The following **transposition (etc) identities** are useful [2 comp version in square brackets]:

$$(\overline{a^c} P_L b) = (\overline{b^c} P_L a) \qquad [\chi\psi = \psi\chi] \tag{1.22}$$

$$(\overline{a} P_L b)^\dagger = (\overline{b} P_R a) \qquad [(\psi\chi)^\dagger = \overline{\psi}\,\overline{\chi}] \tag{1.23}$$

$$(\overline{a}\gamma^\mu P b)^\dagger = (\overline{b}\gamma^\mu P a) \qquad [(\chi\sigma\overline{\psi})^\dagger = \psi\sigma\overline{\chi}] \tag{1.24}$$

$$(\overline{a^c}\gamma^\mu P_{L,R} b^c) = -(\overline{b}\gamma^\mu P_{R,L} a) \qquad [\chi\sigma\overline{\psi} = -\overline{\psi}\,\overline{\sigma}\chi] \tag{1.25}$$

The **Fierz identities** in four-component notation, are a magic formula (see eg Cheng and Li Appendix):
1) Define $\{\Gamma_X\} = \{1, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu\nu}\}$
2) Write the 4-f vertex whose spinors you wish to reorder as

$$\sum_X g_X (\overline{\psi_1}\Gamma_X \psi_2)(\overline{\psi_3}\Gamma^X \psi_4) = \sum_Y \hat{g}_Y (\overline{\psi_1}\Gamma_Y \psi_4)(\overline{\psi_3}\Gamma^Y \psi_2)$$

3) The Fierz identities say (this version includes -ve signs for spinor anti-comm.)

$$\begin{pmatrix} \hat{g}_S \\ \hat{g}_V \\ \hat{g}_T \\ \hat{g}_A \\ \hat{g}_P \end{pmatrix} = -\frac{1}{4} \begin{bmatrix} 1 & 4 & 12 & -4 & 1 \\ 1 & -2 & 0 & -2 & -1 \\ \frac{1}{2} & 0 & -2 & 0 & \frac{1}{2} \\ -1 & -2 & 0 & -2 & 1 \\ 1 & -4 & 12 & 4 & 1 \end{bmatrix} \begin{pmatrix} g_S \\ g_V \\ g_T \\ g_A \\ g_P \end{pmatrix} \tag{1.26}$$

Ex:

$$(\overline{a} P_L b)(\overline{c} P_R d) = -\frac{1}{2}(\overline{a}\gamma^\mu P_R d)(\overline{c}\gamma_\mu P_L b) \tag{1.27}$$

$$(\overline{a}\gamma^\mu P_{L,R} b)(\overline{c}\gamma_\mu P_{L,R} d) = (\overline{a}\gamma^\mu P_{L,R} d)(\overline{c}\gamma_\mu P_{L,R} b) \tag{1.28}$$

1.4.1 Fierz identities: where did they come from?

Appendix B: Wess+Bagger
Peskin+Schroeder 3.4
Nieves+Pal hep-ph/0306087

Makes more sense in 2-comp notation, see Peskin+Schroeder, sec 3.4, using Pauli matrix identities. For instance, the SU(2) generator identity:

$$\frac{1}{4}\sum_a [\sigma^a]_{ij}[\sigma^a]_{kl} = 2(\delta_{il}\delta_{kj} - \frac{1}{2}\delta_{ij}\delta_{kl}) \tag{1.29}$$

can be rewritten, in 2 comp notn:

$$[\sigma^\mu]_{\alpha\overline{\beta}}[\overline{\sigma}_\mu]^{\overline{\eta}\rho} = 2\delta_\alpha^\rho\delta_{\overline{\beta}}^{\overline{\eta}}$$

Then, recalling the respective up-down (down-up) contraction rules for unbarred (barred) indices, one obtains

$$(\chi\sigma^\mu\overline{\eta})(\overline{\psi}\overline{\sigma}_\mu\rho) = -2(\chi^\alpha\rho_\alpha)(\overline{\psi}_{\overline{\beta}}\overline{\psi}^{\overline{\beta}})$$

which gives (1.27) in 4-component notn:

$$(\overline{a}P_L b)(\overline{c}P_R d) = -\frac{1}{2}(\overline{a}\gamma^\mu P_R d)(\overline{c}\gamma_\mu P_L b) \tag{1.30}$$

Then, eqn (1.29) can also be used to write

$$[\sigma^\mu]_{\alpha\overline{\beta}}[\sigma_\mu]_{\rho\overline{\eta}} = 2(\delta_{\alpha\overline{\eta}}\delta_{\overline{\beta}\rho} - \delta_{\alpha\overline{\beta}}\delta_{\overline{\eta}\rho}) \quad .$$

The RHS flips sign under interchange $\overline{\beta} \leftrightarrow \overline{\eta}$, so

$$[\sigma^\mu]_{\alpha\overline{\beta}}[\sigma_\mu]_{\rho\overline{\eta}} = -[\sigma^\mu]_{\alpha\overline{\eta}}[\sigma_\mu]_{\rho\overline{\beta}}$$

and including another -ve sign for anti-commuting Grassman fields(see P+S after eqn 18.43), gives

$$(\overline{a_R}\gamma^\mu d_R)(\overline{c_R}\gamma_\mu b_R) = (\overline{a_R}\gamma^\mu b_R)(\overline{c_R}\gamma_\mu d_R) \tag{1.31}$$

Recall $\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{bmatrix}$

1.4.2 Fierz identities: proving the magic formula

Peskin + Schroeder, problem 3.6

1. For given 4-component spinors $\psi_1, \psi_2, \psi_3, \psi_4$ and (hermitian?) matrices A, B, it is always possible to write
$$(\overline{\psi_1} R \psi_2)(\overline{\psi_3} S \psi_4) = (\overline{\psi_1} M \psi_4)(\overline{\psi_3} N \psi_2)$$
by defining $R_{ij} S_{kl} = -M_{il} N_{kj}$ (-ve sign because three fermion anticommutations). The Fierz identities are relations of this form, where A, B, C, D where expanded on a convenient basis of matrices
2. Recall that a convenient basis of 4*4 matrices is $\{\Gamma_A\} = \{1, i\gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu,\nu}\}$ There are 16; they satisfy $\Gamma_A^\dagger = \gamma_0 \Gamma_A \gamma_0$ according to Appendix D of Haber+Kane Phys Rep. (NB the $i\gamma_5$). Any matrix M can be written as a sum with complex coefficients: $M = \sum_A \Gamma_A tr(\Gamma^A M)$.
3. to normalise this basis, check that $tr(\Gamma_A^\dagger \Gamma_B) = 4\delta_{AB}$.
4. Suppose $R = S = \Gamma_X$. Then
$$[\Gamma_X]_{ij}[\Gamma_X]_{kl} = -M_{il}N_{kj} = -\sum_{C,D} m^C n^D [\Gamma_C]_{il}[\Gamma_D]_{kj} = -\sum_{C,D} C_{XX}^{CD}[\Gamma_C]_{il}[\Gamma_D]_{kj}$$
Mutiply both sides by $[\Gamma_Y^\dagger]_{li}[\Gamma_Z^\dagger]_{jk}$ and sum on i, j, k, l, gives:
$$C_{XXCD} = -\sum_{C,D} tr(\Gamma_X \Gamma_C^\dagger \Gamma_X \Gamma_D^\dagger)$$
5. remember to remove the i from γ_5 (also, always check Γ^X defns before using Fierz trans)
6. notice there is an issue about an overall sign in Fierz transformations, which I interpret as the sign from anti-comm of Grassman fields. I beleive the sign of (1.27) — its in Wess+Bagger— it agrees with Cheng+Li, but is overall opposite from Nieves and Pal. It also agrees with Peskin+Schroeder, see eqn (1.31).

1.4.3 things to note in processing 4-f vertices

- there is no negative sign from interchanging fermions when one takes the hermitian conjugate of a vertex. This is because the definition of conjugation for Grassman numbers is like for operators: $(\eta\theta)^* = \theta^*\eta^*$ (see Peskin 9.65). So : $(\chi\psi)^* = (\chi^\alpha\psi_\alpha)^* = \psi^*_\alpha\chi^{\alpha *} \equiv \overline{\psi}\overline{\chi} = \overline{\chi}\overline{\psi}$ [Notice also that the grassman components of a fermion anti-commute, but the product is also antisymmetric: $\chi\psi = \chi^\alpha\psi_\alpha = \chi_\beta\epsilon^{\alpha\beta}\psi_\alpha = \psi^\alpha\chi_\alpha = \psi\chi$.]

- the third component of the scalar bilepton triplet induces four-fermion vertices $(\overline{e^c}P_L\nu)(\overline{\nu}P_Re^c)$, $(\overline{e^c}P_L\nu)\times(\overline{e}P_R\nu^c)$, $(\overline{\nu^c}P_Le)(\overline{e}P_R\nu^c)$, $(\overline{\nu^c}P_Le)(\overline{\nu}P_Re^c)$. These are all the same. This is obvious in 2 component notation, and one can show in Dirac notn that $\overline{a^c}Pb = \overline{b^c}Pa$. These 4 effective vertices Fierz transform into equivalent $V \pm A$ vertices: $(\overline{e^c}\gamma^\mu P_Re^c)(\overline{\nu}\gamma_\mu P_L\nu)$, $(\overline{e^c}\gamma^\mu R\nu^c)(\overline{e}\gamma_\mu L\nu)$, $(\overline{\nu^c}\gamma^\mu Re^c)(\overline{\nu}\gamma_\mu Le)$, $(\overline{\nu^c}\gamma^\mu R\nu^c)(\overline{e}\gamma_\mu Le)$. These are all equivalent to $-(\overline{\nu}\gamma^\mu L\nu)(\overline{e}\gamma_\mu Le)$, as required. This is because $V^2 + A^2 \rightarrow V^2 + A^A$ under a Fierz Transformation, so $(\overline{e}\gamma^\mu L\nu)(\overline{\nu}\gamma_\mu Le) = (\overline{e}\gamma_\mu Le)(\overline{\nu}\gamma_\mu L\nu)$.

- Note that the only way to generate matrix elements of the form $(\overline{\psi}\sigma^{\mu\nu}\chi)(\overline{\epsilon}\sigma_{\mu\nu}\delta)$ from the exchange of a scalar or vector particle is by Fierz-rearranging a scalar induced four fermion vertex where the scalar coupled to left-handed fermions at one end, and right-handed at the other (operators of the form $(\overline{\psi}R\chi)(\overline{\epsilon}R\delta)$).

1.5 Back to S_o with chiral coupling λ_{LS_o} in π decay

So now rewrite our contact interactions (using eqns 1.23,1.27 and 1.25)

$$\begin{aligned}
-\frac{|\lambda_{LS_o}|^2}{m_S^2}(\overline{(d_L)^c}\nu_L)(\overline{(u_L)^c}e_L)^\dagger &= -\frac{|\lambda_{LS_o}|^2}{m_S^2}(\overline{(d_L)^c}\nu_L)(\overline{e_L}P_R(u_L)^c) \\
&= \frac{|\lambda_{LS_o}|^2}{2m_S^2}(\overline{(d_L)^c}\gamma^\mu P_R(u_L)^c)(\overline{e_L}\gamma_\mu P_L\nu_L) \\
&= -\frac{|\lambda_{LS_o}|^2}{2m_S^2}(\overline{u_L}\gamma^\mu P_L d_L)(\overline{e_L}\gamma_\mu P_L\nu_L) \qquad (1.32)
\end{aligned}$$

Comparing to eqn (1.19), S_o with coupling λ_{LS_o} induces a shift

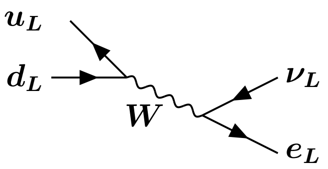
$$\frac{4G_F}{\sqrt{2}} \to \frac{4G_F}{\sqrt{2}} + \frac{|\lambda_{LS_o}|^2}{2m_S^2} \equiv \frac{4G_F}{\sqrt{2}}(1+\epsilon)$$

If suppose no NP in G_F determination from μ decay, then can constrain ϵ for $\pi \to e\bar{\nu}$, or $\pi \to \mu\bar{\nu}$, to fit in the experimental errors.

Exchange of a leptoquark induced a contact interaction also induced by W exchange ! (Of course, the W and S_o can respectively induce other contact interactions that might be different) So from the perspective of an low energy theory, maybe don't need the details of the high energy model? Maybe can describe generic heavy NP by a generic collection of contact interactions?

*Here, the renormalisable low energy theory is QED*QCD for quarks and charged leptons, + non-interacting neutrinos. The interactions mediated by heavier particles (W, S_o, etc) can be described as contact interactions respecting QED and QCD gauge invariance, and Lorentz invariance.*

1.6 Problem : calculate the decay rate of the pion + constrain the leptoquark



1. take contact interaction approx to W propagator (unitary gauge)

$$\frac{-i}{q^2 - m_W^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m_W^2}\right) \longrightarrow \frac{ig^{\mu\nu}}{m_W^2}$$

2. but the quarks are not free...rather than free spinors for the u and d quarks, use that $\langle 0|\overline{q}\gamma^\mu\gamma^5 q|\pi\rangle = f_\pi P^\mu$. That is, replace in usual Feynman rules:

$$\bar{v}(p_u)\gamma^\alpha P_L u(p_d) \to \frac{f_\pi}{2} P_\pi^\alpha$$

3. Then use 2 body decay rate (without neglecting the mass of the final-state-charged-lepton)

$$\Gamma(\pi^- \to \overline{\nu_e} e) = \frac{1}{2m_\pi}\int \frac{d^3p_e}{(2\pi)^3 2p_e^0}\frac{d^3k_e}{(2\pi)^3 2k_e^0}|\mathcal{M}|^2(2\pi)^4\delta^4(P_\pi - p_e - k_e)$$

...to obtain:

$$\Gamma\left(\pi \to e\bar{\nu}\right) = \frac{G_F^2 f_\pi^2 m_\pi^3}{2\pi}\frac{(m_\pi^2 - m_e^2)^2}{m_\pi^4}\frac{m_e^2}{m_\pi^2}$$

In PDG, $BR(\pi^+ \to e^+\nu) = 1.230 \pm 0.004 \times 10^{-4}$, agrees with SM expectation, so at "two σ", constrain

$$\epsilon \lesssim 2 \times \frac{4}{1230}$$

1.8 And what if S_o has couplings λ_{LS_o} and λ_{RS_o}

$$\begin{aligned} -\frac{\lambda_{LS_o}\lambda^*_{RS_o}}{m_S^2}(\overline{(d_L)^c}\nu_L)(\overline{(u_R)^c}e_R)^\dagger + h.c. &= -\frac{\lambda_{LS_o}\lambda^*_{RS_o}}{m_S^2}(\overline{(d_L)^c}\nu_L)(\overline{e_R}P_L(u_R)^c) + h.c. \\ &= -\frac{\lambda_{LS_o}\lambda^*_{RS_o}}{m_S^2}(\overline{(\nu_L)^c}d_L)(\overline{(u_R)}P_L(e_R)^c) + h.c. \\ &= \frac{\lambda_{LS_o}\lambda^*_{RS_o}}{2m_S^2}(\overline{(\nu_L)^c}(e_R)^c)(\overline{(u_R)}d_L) + \ tensor + h.c. \end{aligned} \tag{1.38}$$

where used the first transposition identity and Fiertz. A “pseudoscalar” contact interaction! (lets forget tensor). The pion is pseudoscalar — maybe could decay directly to $e_R\bar{\nu}$ without chirality flip for e via m_e?
[**The general meson decay formula:** Can show that in the presence of contact interactions of the form (flavours i, j, k, n summed)

$$-\epsilon^{ijkn}_{(1)\ell q}\frac{4G_F}{\sqrt{2}}(\bar{\ell}^i\gamma^\mu P_L\ell^j)(\bar{q}^k\gamma_\mu P_L q^n) - \left\{\epsilon^{ijkn}_{qde}\frac{4G_F}{\sqrt{2}}(\bar{e}^i P_L\ell^j)(\bar{q}^k P_R d^n) + h.c.\right\}$$

the decay rate of a pseudoscalar meson M, made of $(\bar{q}_k d_n)$, is

$$\begin{aligned} \Gamma\left(M_{kn} \to l^i\bar{l}^j\right) &= \frac{kG_F^2}{\pi m_M^2}\Big\{ \left[\epsilon^{ijkn}_{(1)\ell q}\right]^2 \widetilde{A}^2\left[(m_M^2 - m_i^2 - m_j^2)(m_i^2 + m_j^2) + 4m_i^2m_j^2\right] + \\ &\qquad (\epsilon^{ijkn}_{qde})^2\widetilde{P}^2\left(m_M^2 - m_i^2 - m_j^2\right) + 2\left[\epsilon^{ijkn}_{qde}\epsilon^{ijkn}_{(1)\ell q}\right]\widetilde{A}\widetilde{P}m_j\left(m_M^2 + m_i^2 - m_j^2\right)\Big\} \end{aligned}$$

where the axial-vector and pseudoscalar matrix elements are:

$$\widetilde{A}P^\mu = \frac{1}{2}\langle 0|\overline{q}\gamma^\mu\gamma^5 q|M\rangle = \frac{f_M P^\mu}{2} \qquad \widetilde{P} = \frac{1}{2}\langle 0|\overline{q}\gamma^5 q|M\rangle = \frac{f_M m_M}{2}\frac{m_M}{m_k + m_n} \quad .$$

P^μ is the momentum of the meson, and k is the magnitude of the lepton 3-momentum in the center-of-mass frame:

$$k^2 = \frac{1}{4m_M^2}\left[\left(m_M^2 - (m_i + m_j)^2\right)\left(m_M^2 - (m_i - m_j)^2\right)\right]$$

1.9 Problem: pseudoscalar meson decay bounds on a leptoquark coupling to L and R fermions

compute pion decay rate due to S_o with $\lambda_L \sim 1 \sim \lambda_R$ couplings. What is the lower bound on the S_o mass? Compare to the bound when $\lambda_L \sim 1, \lambda_R = 0$.

1.10 Approximating leptoquark exchange as a contact interaction, and $Q^2 \ll m_{LQ}^2$

The renormalisable leptoquark interactions respect SM gauge symmetries, and their masses do too. So the (non-renormalisable!) contact interactions which the leptoquarks generate at energies $\ll m_{LQ}^2$, should also respect SM gauge symmetries. Therefore, at low energies, a given leptoquark will induce a specific list of SM gauge invariant lepton-antilepton-quark-antiquark interactions. These "four-fermion" operators/contact interactions are of mass dimension six, $\propto 1/m_{LQ}^2$.

So rather than study all possible leptoquarks and combinations of leptoquarks, one could instead study a generic set of contact interactions with arbitrary coefficients. They are on the next page... This more model-independent approach is called Effective Field Theory.

Curiosity: the $SU(2) \times U(1)$ symmetry of the SM is broken by the Higgs vev. Nonetheless, at tree level, W and Z exchange induce SU(2) gauge-invariant operators Consider the four-lepton operator

$$\left(\bar{\ell}_e \tau^I \gamma^\mu P_L \ell_e\right) \left(\bar{\ell}_\mu \tau^I \gamma_\mu P_L \ell_\mu\right),$$

which would be generated by the exchange of a massive SU(3) triplet vector boson.

1.11 Problem: weak gauge boson exchange generates SM-gauge-invariant operators

Starting from the SM Lagrangian in the PDB, check that W and Z exchange between the electron and muon doublets induce this operator. Why might this be the case? reponse, I think: SU(2) is a gauge sym, so its always there. Just broken by the Higgs vev. So should be able to describe this breaking, in the effective theory, with SU(2) invar operators involving the Higgs field.

1.12 A list of dimension six, 2-quark-2-lepton operators, SM gauge invariant

The list of $S(3) \times SU(2) \times U(1)$ invariant two-lepton two-quark operators, with lorentz structure $\overline{L}L\overline{L}L$, $\overline{R}R\overline{R}R$ or $\overline{L}R\overline{R}L$, singlet or triplet SU(2) gauge contractions (described in the operator subscript), and all possible inequivalent flavour index combinations:

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$$\begin{aligned}
\mathcal{O}^{ijkn}_{(1)\ell q} &= \left(\bar{\ell}_i \gamma^\mu P_L \ell_j\right) \left(\bar{q}_k \gamma_\mu P_L q_n\right), \\
&= \left(\overline{\nu}_i \gamma^\mu \nu_j + \bar{e}_i \gamma^\mu P_L e_j\right) \left(\bar{u}_k \gamma_\mu P_L u_n + \bar{d}_k \gamma_\mu P_L d_n\right), \qquad (1.39)\\
\mathcal{O}^{ijkn}_{(3)\ell q} &= \left(\bar{\ell}_i \tau^I \gamma^\mu P_L \ell_j\right) \left(\bar{q}_k \tau^I \gamma_\mu P_L q_n\right), \\
&= 2\ (\overline{\nu}_i \gamma^\mu P_L e_j)(\overline{d}_k \gamma_\mu P_L u_l) + 2(\overline{e}_i \gamma^\mu P_L \nu_j)(\overline{u}_k \gamma_\mu P_L d_l) \\
&\quad + \Big[(\overline{\nu}_i \gamma^\mu P_L \nu_j)(\overline{u}_k \gamma_\mu P_L u_l) + (\overline{e}_i \gamma^\mu P_L e_j)(\overline{d}_k \gamma_\mu P_L d_l) \\
&\quad - (\overline{\nu}_i \gamma^\mu P_L \nu_j)(\overline{d}_k \gamma_\mu P_L d_l) - (\overline{e}_i \gamma^\mu P_L e_j)(\overline{u}_k \gamma_\mu P_L u_l)\Big], \qquad (1.40)\\
\mathcal{O}^{ijkn}_{eq} &= \left(\bar{e}_i \gamma^\mu P_R e_j\right) \left(\bar{q}_k \gamma_\mu P_L q_n\right), \\
\mathcal{O}^{ijkn}_{ed} &= \left(\bar{e}_i \gamma^\mu P_R e_j\right) \left(\bar{d}_k \gamma_\mu P_R d_n\right), \\
\mathcal{O}^{ijkn}_{eu} &= \left(\bar{e}_i \gamma^\mu P_R e_j\right) \left(\bar{u}_k \gamma_\mu P_R u_n\right), \\
\mathcal{O}^{ijkn}_{\ell u} &= \left(\bar{\ell}_i \gamma^\mu P_L \ell_j\right) \left(\bar{u}_k \gamma_\mu P_R u_n\right) \\
\mathcal{O}^{ijkn}_{ld} &= \left(\bar{\ell}_i \gamma^\mu P_L \ell_j\right) \left(\bar{d}_k \gamma_\mu P_R d_n\right) \qquad (1.41)
\end{aligned}$$

where the Lorentz, colour and SU(2) are contractions are implicit within the parentheses, and τ^I are the Pauli matrices ($\tau^3 = diag\{1, -1\}$). With the same notation (but now SU(2) contraction between the parentheses), the S±P operators are:

$$\begin{aligned}
\mathcal{O}^{ijkn}_{\ell q S} &= \left(\bar{\ell}_i e_j\right) \left(\bar{q}_k u_n\right), \\
\mathcal{O}^{ijkn}_{qde} &= \left(\bar{\ell}_i e_j\right) \left(\bar{d}_k q_n\right). \qquad (1.42)
\end{aligned}$$

1.12.1 Flavour sums, 2s and $+h.c$ in the Lagrangian

The operators can be added to the SM Lagrangian with predicted, or arbitrary, coefficients (of mass dimension -2). For instance, to give a general description of 2-lepton, 2quark contact interactions (in the most general case, maybe there are tensor interactions to include too?), they can all be included in the Lagrangian, summed on flavour. For the first six operators, one can write:

$$\delta\mathcal{L} = \sum_{i,j,k,n=1}^{3} \frac{C_I^{ijkn}}{\Lambda^2} \mathcal{O}_I^{ijkn}$$

where the Cs are dimensionless, Λ is some high mass scale, and, NB, the flavour sums are from 1..3 but there is *no* $+.h.c.$. One can convince oneself that the conjugate operators are included in the flavour sum; this means that the coefficients satisfy $C_I^{ijkn} = C_I^{jinn\,*}$). For the last two operators of eqn (1.42), the flavour sums also run 1..3, but one must in addition *include the* $+h.c.$.

1.13 contraints from the PDB*lattice

The PDB contains a long list of observed or bounded branching ratios for the decays of mesons of different flavours. The observed decay rates tend to agree with the SM expectations, when these can be reliably calculated. The decays with final state leptons can therefore be used to constrain the contact interactions induced at tree-level by leptoquarks.

We obtained some bounds from the decay of the pion. The general recipe can be extended to other mesons. For instance, to constrain a contact interaction $(\overline{d}\gamma^\alpha P_L s)(\overline{e}\gamma^\alpha P_L \mu)$ from the decay $K \to \pi e \overline{\mu}$:

1. obtain the matrix element for the quark current between the initial and final meson states. These can be obtained
    (a) from the lattice
    Then you calculate the LQ-induced partial decay rate, and compare to the measured total decay rate to obtain a bound
    (b) by comparing to a "similar" process induced in the SM by a "similar" quark current, with more or less sophisticated corrections (eg isospin)

$$\langle \pi^+ | (\overline{d}\gamma^\alpha P_L s) | K^+ \rangle \simeq \sqrt{2} \langle \pi^0 | (\overline{u}\gamma^\alpha P_L s) | K^+ \rangle$$

    Then there is no need to compute a rate: can constrain the coefficient of the LQ contact interaction $<$ SM contact interaction $\times \sqrt{\text{ratio of BRs}}$.

2. when the LQ induces a process also present in the SM, should take some care about possible interference and signs, and can get a bound from the allowed deviations of the data from the SM prediction(means you need to be able to *predict* the SM rate).

Notice this recipe is very sloppy — it sets bounds on LQ processes "one at a time". So for instance, if one assumes that muon decay is mediated by the SM so one can extract G_F, then one can use CKM unitarity and β decay to constrain LQ-induced contact interactions. But if there was also NP in muon decay, these bounds would not be correct.

??Ideally, one should simultaneously fit all coefficients of all contact interactions to all data??

1.14 μ-e conversion and the leptoquark

μ-e conversion starts by of shooting a beam of μ^- at a nuclear target. The μ^- is captured by a nucleus, and tumbles down the atomic levels (mostly ends up in the 1s state). Then one looks for an emitted e^- of energy = [muon mass - binding energy]. Several new experiments are under discussion, with hopes of reaching a sensitivity as low as $\Gamma(\mu \to e) \sim 10^{-18}\Gamma_{SM}(\text{capture})$.

In the SM, the μ decays to a neutrino by β capture on the nucleus. The μ wavefunction has significant overlap with the nucleus (common targets aretitanium with $Z = 22$ and $\simeq 48$ nucleons, and gold with $Z = 79$ and $\simeq 197$ nucleons), so this contact interaction is possible, and can be detected because the final nucleus is different.

Rough bounds on leptoquark-induced contact interactions with the flavour structure (Γ some Lorentz structure)

$$\epsilon\frac{4G_F}{\sqrt{2}}(\overline{u}\Gamma^A P_X u)(\overline{e}\Gamma^A P_Y \mu) \quad , \quad \epsilon\frac{4G_F}{\sqrt{2}}(\overline{d}\Gamma^A P_X d)(\overline{e}\Gamma^A P_Y \mu)$$

can be obtained as

$$\epsilon \lesssim \sqrt{\frac{\Gamma(\mu A \to eA)}{\Gamma_{SM}(\text{capture})}} \sim 10^{-6}$$

in practise, the bounds on the conversion rate depend on the Lorentz structure of the matrix element (the conversion process can be coherent, unlike capture), and on the nucleus (the μ is more or less *in* the nucleus, etc). See eg , Kuno+Okada, Rev.Mod.Phys. 73(2001) 151[9909265]; Kitano etal[0203110]; Czarnecki etal[9801218].