4 The type I seesaw

4.1 Some curious features of Dirac neutrino masses

Puzzle 1: if the observed neutrino masses are Dirac : $m\overline{\nu_L}\nu_R + hc$, why are neutrino Yukawa eigenvalues $\ll$ other fermions?

Some solutions:

- in SUSY, put a symmetry to forbid as an F-term. Appears in SUGRA as a D-term $\propto m_{SUSY}/m_{pl}$. ArkaniHamed Borzumati
- in extra dimensions, ν_R and ν_L live in different places: little overlap. Grossman+Neubert...
- Ignore this puzzle: we don't understand Yukawas
- ...

Puzzle 2: ν_R is gauge singlet, why does it not have a majorana mass? (not forbidden by SM gauge symmetries...)

A solution:

- Put a symmetry. Such as lepton number L, or $B - L$.

Another solution

- Put a mass...

4.2 The Type I seesaw

4.2.1 One generation

Adding a gauge-singlet/right-handed/sterile ν_R (or sometimes called N) allows "Dirac" masses for νs:

$$\mathcal{L}_{lep}^{Yuk} = -y_e(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} \Phi_+ \\ \Phi_0 \end{pmatrix} e_R \ + \ \lambda(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} -\Phi_0^* \\ \Phi_- \end{pmatrix} \nu_R + h.c.$$

But the ν_R has no gauge interactions, so its allowed a mass:

$$\mathcal{L}_{lep}^{Yuk} = -y_e(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} \Phi_+ \\ \Phi_0 \end{pmatrix} e_R \ + \ \lambda(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} -\Phi_0^* \\ \Phi_- \end{pmatrix} \nu_R \ - \ \frac{M}{2}\overline{\nu_R^c}\nu_R + h.c.$$

Adding a right-handed (sterile) ν_R with all renorm. interactions:

$$\mathcal{L}^{Yuk}_{lep} = -y_e(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} \Phi_+ \\ \Phi_0 \end{pmatrix} e_R \; + \; \lambda(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} -\Phi_0^* \\ \Phi_- \end{pmatrix} \nu_R \; - \; \frac{M}{2}\overline{\nu_R^c}\nu_R + h.c.$$

$$- m_e \overline{e_L} e_R \qquad\qquad -m_D \overline{\nu_L}\nu_R \qquad\qquad -\frac{M}{2}\overline{(\nu_R)^c}\nu_R + h.c.$$

$\Rightarrow$ neutrino mass matrix:

$$\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \begin{bmatrix} 0 & m_D \\ m_D & M \end{bmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} \qquad (\nu_L^c \equiv (\nu_L)^c)$$

$\Rightarrow$ 4-component "majorana" eigenvectors ν_m, n, such that $P_L \nu_m \simeq \nu_L$ with mass $m_\nu \sim \frac{m_D^2}{M}$, $P_R n \simeq \nu_R$ with mass $\sim M$. More precisely, for $M \gg \lambda v$:

$$\text{eigenval.} = \frac{1}{2}\left\{M \pm \sqrt{M^2 + 4|\lambda|^2 v^2}\right\} \simeq M + \frac{|\lambda|^2 v^2}{M}, -\frac{|\lambda|^2 v^2}{M} \equiv M(1+\varepsilon^2), -\varepsilon^2 M \qquad , \quad \varepsilon = \frac{|\lambda| v}{M}$$

(so trace remains M, as should be), and the right-handed eigenvectors are

$$P_R \begin{pmatrix} \nu_m \\ n \end{pmatrix} = \frac{1}{\sqrt{1+\epsilon^2}} \begin{bmatrix} 1 & -\epsilon \\ \epsilon & 1 \end{bmatrix} \begin{pmatrix} \nu_L^c \\ N \end{pmatrix} \qquad , \qquad \epsilon = \frac{|\lambda| v}{M(1+\varepsilon^2)} \tag{4.47}$$

so at order ϵ^2, I can take $\varepsilon = \epsilon$.

We will be interested to calculate to $\mathcal{O}(\epsilon^2)$, because the massive light ν contribution obtained in the Dirac case was $\mathcal{O}(m_\nu^2)$, which in the Majorana case means $\mathcal{O}(M^{-2})$, so we need all terms of $\mathcal{O}(M^{-2})$.

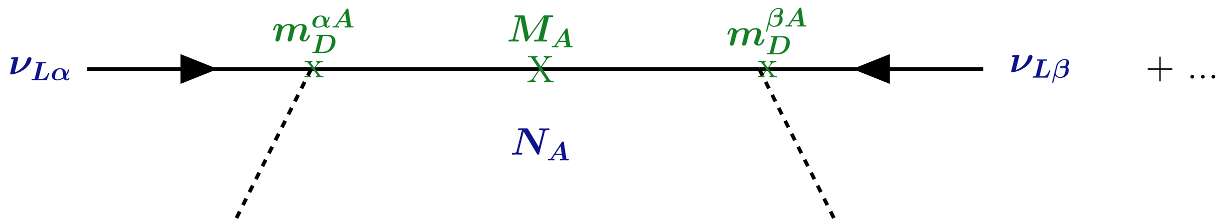
4.2.2 The Type 1 See-Saw in three generations

• in the charged lepton ("flavour") and singlet ν_R mass bases, at large energy scale $\gg M_i$:

$$\mathcal{L} \;=\; \mathcal{L}_{SM} + \lambda^*_{\alpha J}\overline{\ell}_\alpha \cdot H N_J - \frac{1}{2}\overline{N_J} M_J N_J^c$$

21 parameters chez les leptons:
$m_e, m_\mu, m_\tau, M_1, M_2, M_3$
18 - 3 (ℓ phases) in λ

• at energy scales $\ll M$, get effective lepton number violating "contact interaction" $(\overline{\ell^c} H)(H\ell)$ of mass dimension 5, which, with EWSB, gives majorana masses to doublet neutrinos:



$$\lambda M^{-1} \lambda^{\mathbf{T}} \langle H^0 \rangle^2 \;=\; [m_\nu] \;=\; U^* D_m U^\dagger$$

12 parameters:
$m_e, m_\mu, m_\tau, m_1, m_2, m_3$
6 in U_{MNS}

• notice that U is the unitary PMNS matrix, transforming between charged lepton and doublet neutrino mass eigenstates. The mixing of doublets and singlets, which induces non-unitarity, is not present at order $1/M$ in an effective theory without the singlets!

4.3 $\mu \to e\gamma$ in the type 1 seesaw

It is often said that the caln of $\mu \to e\gamma$ for Dirac neutrino masses applies for Majorana masses as well, because the calculation done for Dirac would be the same with Majorana masses. This is true. *BUT*, Majorana masses only can arise from a non-renormalisable operator, so the NP responsable for that operator could give other contributions to $\mu \to e\gamma$ which could be more or less important. We want to see the other contributions for the type I seesaw.

The aim is to understand, from a bottom-up effective field theory perspective, why the light mass contribution is insufficient (is that obvious? Is it model-dependent?), and whether there are other low-energy traces of the other contributions.

4.3.1 A toy model

Suppose add one singlet fermion N, with majorana mass M, to the two generation SM. Leptonic Lagrangian will be (for H the SM Higgs, and SUSY conventions with $*$ on Yukawas, so superpotential $y_e^* LHE^c$)

$$\mathcal{L} = \lambda_\alpha \overline{\ell}_\alpha H^{c*} N - \frac{M}{2} NN - y_\alpha \overline{\ell}_\alpha H^* e_\alpha + h.c. \tag{4.48}$$

$$= \lambda_\alpha (\overline{\nu_\alpha}, \overline{e_\alpha})_L \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} -H^- \\ H^0 \end{pmatrix} N - \frac{M}{2} NN - y_\alpha (\overline{\nu_\alpha}, \overline{e_\alpha})_L \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} H^{0*} \\ H^{-*} \end{pmatrix} e_{R\alpha} + h.c.$$

$$= -\lambda_\alpha \overline{\nu_{\alpha L}} H^0 N_R - \lambda_\alpha \overline{e_{\alpha L}} H^- N_R - \frac{M}{2} NN - y_\alpha \overline{e_{\alpha L}} H^{0^*} e_{R\alpha} + y_\alpha \overline{\nu_{\alpha L}} H^+ e_{R\alpha} + h.c. \tag{4.49}$$

for $\alpha = e, \mu$.

4.4 Calculating $\mu \to e\gamma$ in the mass basis, with the complete particle content

4.4.1 Lagrangian

After EWSB, the mass terms become

$$\mathcal{L}_{mass} \rightarrow -\lambda_\alpha \overline{\nu_{\alpha L}} H^0 N_R - \frac{M}{2} NN - y_\alpha \overline{e_{\alpha L}} H^{0^*} e_{R\alpha} + h.c. \tag{4.50}$$

$$= -|\lambda| v \overline{\nu_2} N_R - \frac{M}{2} NN - m_\alpha \overline{e_{L\alpha}} e_{R\alpha} + h.c. \tag{4.51}$$

for $\alpha = e, \mu$.

The massive doublet neutrino is ν_2:

$$\nu_2 \equiv \frac{\lambda_e}{|\lambda|}\nu_e + \frac{\lambda_\mu}{|\lambda|}\nu_\mu = \sin\theta \nu_e + \cos\theta \nu_\mu \quad , \quad |\lambda| = \sqrt{\lambda_e^2 + \lambda_\mu^2} \tag{4.52}$$

and the unitary mixing matrix $U_{\alpha i}$ between the mass eigenbasis of ν_i and e_L is

$$U = \begin{bmatrix} c & s \\ -s & c \end{bmatrix}$$

Recall the Majorana masses are

$$M(1+\varepsilon^2), -\varepsilon^2 M \qquad , \quad \varepsilon = \frac{|\lambda| v}{M}$$

and the right-handed components of the Majorana mass eigenstates ν_m, n are

$$P_R \begin{pmatrix} \nu_m \\ n \end{pmatrix} = \frac{1}{\sqrt{1+\epsilon^2}} \begin{bmatrix} 1 & -\epsilon \\ \epsilon & 1 \end{bmatrix} \begin{pmatrix} \nu_2^c \\ N \end{pmatrix} \quad , \quad \begin{pmatrix} \nu_2^c \\ N \end{pmatrix} = \frac{1}{\sqrt{1+\epsilon^2}} \begin{bmatrix} 1 & \epsilon \\ -\epsilon & 1 \end{bmatrix} P_R \begin{pmatrix} \nu_m \\ n \end{pmatrix} \quad , \quad \epsilon = \frac{|\lambda| v}{M(1+\varepsilon^2)} \tag{4.53}$$

Implicitely, on the left in eqn (4.47), I took n and ν_m to be 4-component majorana fermions, so *e.g.* $\nu_2 = \frac{1}{\sqrt{1+\epsilon^2}} P_L(\nu_m + \epsilon n)$

The kinetic terms will be (overall signs consistent with C+L claim that F rules are $i\times$ interaction, and their F rule for W: $ig/\sqrt{2}P_L$, and signs among bosons agrees with conventions of C+L appendix, where have $i\not{D} = i\gamma^\mu(\partial_\mu + eA_\mu)$ for the electron $Q_e = -1$)

$$\begin{aligned}
\mathcal{L}_{KT} &= i\overline{\ell_\alpha}\not{D}\ \ell_\alpha = \overline{\ell_\alpha}(i\not{\partial} + gW^a_\mu\tau_a\gamma^\mu)\ell_\alpha = +\frac{g}{\sqrt{2}}\overline{e_a}\gamma^\sigma W^+_\sigma P_L\nu_\alpha + ... + h.c. \\
&= \Bigg(\frac{cg}{\sqrt{2}}\overline{e_L}\gamma^\sigma W^+_\sigma\nu_1 - \frac{sg}{\sqrt{2}}\overline{\mu_L}\gamma^\sigma W^+_\sigma\nu_1 + h.c. \\
&\quad + \frac{sg}{\sqrt{2(1+\epsilon^2)}}\overline{e_L}\gamma^\sigma W^+_\sigma P_L\nu_m + \frac{cg}{\sqrt{2(1+\epsilon^2)}}\overline{\mu_L}\gamma^\sigma W^+_\sigma P_L\nu_m + h.c. \\
&\quad + \frac{\epsilon sg}{\sqrt{2(1+\epsilon^2)}}\overline{e_L}\gamma^\sigma W^+_\sigma P_L n + \frac{\epsilon cg}{\sqrt{2(1+\epsilon^2)}}\overline{\mu_L}\gamma^\sigma W^+_\sigma P_L n + h.c.\Bigg)
\end{aligned} \tag{4.54}$$

The charged goldstone couplings (need later for F-rules) are

$$\begin{aligned}
-\lambda_\alpha\overline{e_{L\alpha}}H^-N_R &= -s|\lambda|\overline{e_L}H^-P_R(-\epsilon\nu_m + n)/\sqrt{1+\epsilon^2} \\
&\quad - c|\lambda|\overline{\mu_L}H^-P_R(-\epsilon\nu_m + n)/\sqrt{1+\epsilon^2}
\end{aligned} \tag{4.55}$$

4.4.2 diagrams

Notice that we no longer need the wave-fn renormalisation diagrams, because they do not contribute to the dipole.

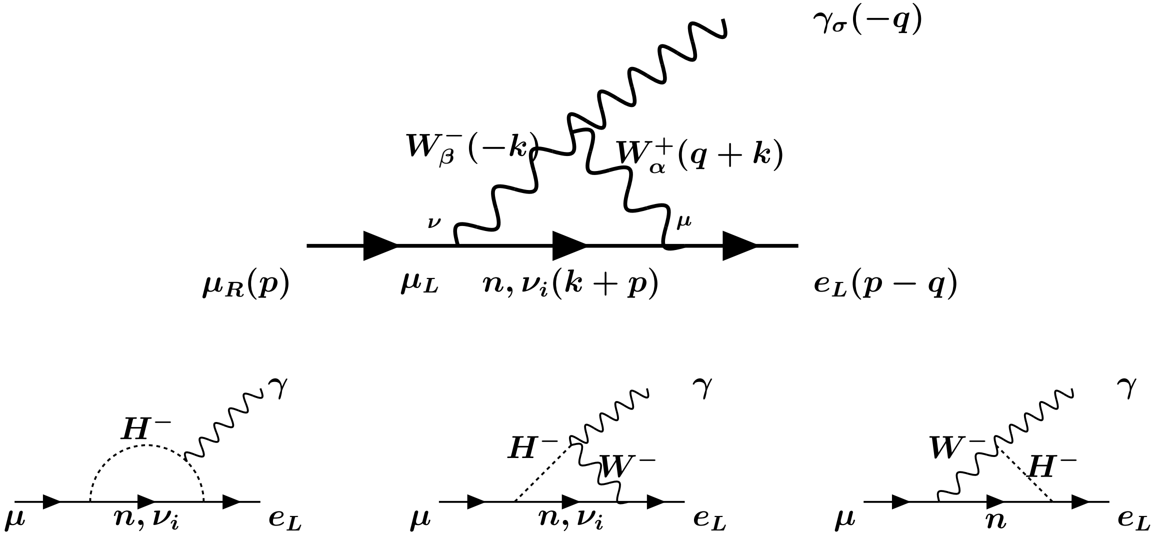


Figure 4.1: One-loop diagrams that contribute to $\mu \to e\gamma$, calculating with broken EW symmetry. H is the goldstone. The vector momenta are ingoing, and $_{\mu,\nu}$ are the Lorentz indices of the γ matrices at the doublet lepton vertices.

4.4.3 Simplifications

- Neglect electron mass
- Neglect any contributions to $\mathcal{M}$ of higher order than $\mathcal{O}(1/M^2)$ (the light mass contribution is this size).
- Feynman 'tHooft gauge (ξ=1)

$$\text{W prop } \propto -i\frac{g_{\mu\nu}}{p^2 - m_W^2} \quad , \quad \text{goldstone prop } \propto i\frac{1}{p^2 - m_W^2}$$

4.4.4 estimates — Feynman 'tHooft gauge, the heavy n

The W diagram: Recall the W propagator in Feynman 'tHooft gauge $\propto g_{\mu\nu}$. And the $n-W$ vertex is $\propto$ the small mixing angle $\epsilon \propto 1/M$, so to get a contribution to $\mathcal{M}(\mu \to e\gamma)$ of $\mathcal{O}(1/M^2)$, need "a quad divergence" inside the loop that cancels the $1/M^2$ of the n propagator.

- The diagram is already multiplied by $\epsilon^2 = |\lambda|^2 v^2/M^2$.
- in the $\gamma^\sigma g_{\sigma\alpha} V^{\alpha\beta\mu} g_{\beta\rho}\gamma^\rho$ contraction, only the the $(p_+ - p_-)\cdot\varepsilon$ term will survive from V ($\gamma_\mu\varepsilon^\mu$ not contribute to mag mo).
- Then there is another upstairs momentum factor from upstairs in the n propagator
- but the mag mo is the coeff of $p\cdot\varepsilon$, so upstairs will have no powers of loop integration momentum k, and downstairs there are three propagators, so convergent integral:

$$\mathcal{M}_{WWn} \simeq (ie)\frac{(ig\epsilon)^2}{2}\overline{u_e}\int\frac{d^4k}{(2\pi)^4}\frac{\gamma^\mu i(\not p + \not k)\gamma_\mu}{(k^2-m_W^2)((k+p)^2-M^2)((k+q)^2-m_W^2)}u_\mu(-i)^2 2k\cdot\varepsilon \tag{4.56}$$

With integration variable shift $k' = k + xp + yq$, and using $\overline{u_e}\not q\, u_\mu = \overline{u_e}(\not p_\mu - \not p_e)u_\mu \simeq \overline{u_e} m_\mu u_\mu$, get

$$\simeq -(-2)e\frac{\epsilon^2 g^2}{2}\overline{u_e}\int\frac{d^4k'}{(2\pi)^4}\int dxdy\frac{\not p(1-x-y)}{[k'^2 - m_W^2(1-x) - xM^2]^3}u_\mu 2xp\cdot\varepsilon \tag{4.57}$$

$$\simeq \frac{e\epsilon^2 g^2 m_\mu}{16\pi^2}(2p\cdot\varepsilon)\overline{u_e}\int dxdy\frac{x(1-x-y)}{[m_W^2(1-x)-xM^2]}u_\mu + \ldots \tag{4.58}$$

$$\sim \mathcal{O}\left(\frac{\epsilon^2}{M^2}\right) \tag{4.59}$$

So in F-'tH gauge ($\xi = 1$), the WWn diagram does not contribute to the $\mathcal{O}(1/M^2)$ matrix element.

The goldstone diagrams:

1) in the two diagrams with a scalar/goldstone and a W, the $W\gamma H$ vertex is $\propto iQ_H e m_W g^{\sigma\alpha}$, so do not contribute at $\mathcal{O}(1/M^2)$: the $\varepsilon_\sigma g^{\sigma\alpha}$ combined with $g_{\alpha\mu}$ of the W propagator gives $\gamma_\sigma\varepsilon^\sigma$, which is a QED vertex renorm.

2) The diagram with two scalar propagators gives

$$\mathcal{M}_{\phi\phi n} = \frac{g^2 sc|\lambda|^2 v^2}{2m_W^2}\overline{u_e}\int \widetilde{dk}\frac{(iP_R)i(\not p + \not k + M)}{(k^2 - m_W^2)((k+p)^2 - M^2)((k+q)^2 - m_W^2)} i(P_L - \frac{m_\mu}{M}P_R)u_\mu(-ie)(-2k-q)\cdot\varepsilon \quad (4.60)$$

Shift the integration variable to $k' = k + xp + yq$, do Feynman reparam, and drop m_μ inside the integral:

$$\begin{aligned}\mathcal{M}_{\phi\phi n} &= esc|\lambda|^2\overline{u_e}\int \widetilde{dk'}\int dxdy\frac{P_R(\not p(1-x) + \not k' - y\not q + M)}{[[k'^2 - (1-x)m_W^2 - xM^2]^3}(P_L - \frac{m_\mu}{M}P_R)u_\mu \times 2(k' - xp)\cdot\varepsilon \quad (4.61)\\ &= -esc|\lambda|^2 2p\cdot\varepsilon\overline{u_e}\int \widetilde{dk'}\int dxdyx\left[\frac{m_\mu(1-x)}{[k'^2 - (1-x)m_W^2 - xM^2]^3} - \frac{m_\mu}{[k'^2 - (1-x)m_W^2 - xM^2]^3}\right]u_\mu\end{aligned}$$

where on second line, extracted the $2p\cdot\varepsilon$. The convergent $\widetilde{dk}$ integration gives

$$\begin{aligned}\mathcal{M}_{\phi\phi n} &= \frac{iesc|\lambda|^2 m_\mu}{16\pi^2}2p\cdot\varepsilon\overline{u_e}\int dxdy\frac{-x^2}{[(1-x)m_W^2 + xM^2]}u_\mu\\ &\to \frac{iesc|\lambda|^2 m_\mu}{16\pi^2}2p\cdot\varepsilon\overline{u_e}\int dx\frac{-x^2}{M^2}u_\mu\end{aligned}$$

which gives

$$\sigma_X \simeq -\frac{1}{3}\frac{iesc|\lambda|^2 m_\mu}{16\pi^2 M^2} \quad (4.62)$$

Compare to Dirac neutrino estimate:

$$m_\mu G_F\frac{e}{16\pi^2}\frac{U_{\mu i}m_{\nu,i}^2 U_{ei}^*}{m_W^2} = \frac{\sqrt{2}}{4v^2}\frac{em_\mu sc|\lambda v|^4}{16\pi^2 m_W^2 M^2} \sim \frac{esc|\lambda|^4 m_\mu}{8\pi^2 g^2 M^2}$$

light neutrino contribution has an additional factor λ^2/g^2! (but maybe $\lambda \gtrsim g$).

4.4.5 estimates — Feynman 'tHooft gauge, the light neutrinos

Recall that for Dirac neutrinos, the leading contribution to LFV vanished due the unitarity of the mixing matrix (this was the GIM mechanism). But now the mixing matrix is no longer exactly unitary (light neutrinos mix with the heavy singlet), so we want to estimate this contribution in our toy model. We do not want to re-discover the light-neutrino-mass contribution we already calculated in the Dirac case, so we neglect the light neutrino masses (but not mixing angles) inside the loop. The sum of the exchange of ν_1 and ν_m will therefore give a factor

$$-sc + \frac{sc}{1-\epsilon^2} = sc\frac{|\lambda|v^2}{M^2} \tag{4.63}$$

The WW diagram:

- in the $\gamma^\sigma g_{\sigma\alpha} V^{\alpha\beta\mu} g_{\beta\rho}\gamma^\rho$ contraction, only the the $(p_+ - p_-)\cdot\varepsilon$ term will survive from V (pcq $\gamma_\mu\varepsilon^\mu$ ne contribue pas a mag mo).
- Then there is another upstairs momentum factor from upstairs in the n propagator, which use to flip chirality on external leg = get m_μ.
- the mag mo is the coeff of $p\cdot\varepsilon$, so upstairs will have no powers of loop integration momentum k, and downstairs there are three propagators, so convergent integral:

$$\mathcal{M}_{WW\nu} \simeq (ie)\frac{(ig)^2}{2}\overline{u_e}\int\frac{d^4k}{(2\pi)^4}\frac{\gamma^\mu i(\not p + \not k)\gamma_\mu}{(k^2-m_W^2)((k+p)^2)((k+q)^2-m_W^2)}u_\mu(-i)^2 2k\cdot\varepsilon \tag{4.64}$$

With integration variable shift $k' = k + xp + yq$, and using $\overline{u_e}\not q\, u_\mu = \overline{u_e}(\not p_\mu - \not p_e)u_\mu \simeq \overline{u_e}m_\mu u_\mu$, get

$$\simeq -(-2)e\frac{g^2}{2}\overline{u_e}\int\frac{d^4k'}{(2\pi)^4}\int dxdy\frac{\not p(1-x-y)}{[k'^2-m_W^2(1-x)]^3}u_\mu 2xp\cdot\varepsilon$$

$$\simeq \frac{eg^2m_\mu}{16\pi^2}(2p\cdot\varepsilon)\overline{u_e}\int dxdy\frac{x(1-x-y)}{[m_W^2(1-x)]}u_\mu + \ldots \sim \frac{eg^2m_\mu}{16\pi^2m_W^2}$$

$$\Rightarrow \sigma_X \sim \frac{esc|\lambda|^2m_\mu}{16\pi^2M^2m_W^2} \tag{4.65}$$

(where mutiplied by eqn (4.63).

The W-goldstone diagrams: vanish in F'tH gauge, because the $W\gamma$goldstone vertex in the last two diagrams is $\propto iQ_\phi e m_W g^{\sigma\alpha}$ (it has a Higgs vev leg), so the $\varepsilon_\sigma g^{\sigma\alpha}$ combined with $g_{\alpha\mu}$ of the W propagator gives $\gamma_\sigma \varepsilon^\sigma$, which is a QED vertex renormalisation.

The goldstone diagram: The goldstone will not couple to ν_1 via λ, because ν_1 is massless. Then e_L couples via λ to the N component of ν_2, which is only a fraction ϵ of ν_2 (recall that we are not interested in contributions to $\mu \to e\gamma$ which are $\propto m_e$, so we cannot couple the goldstone to e_L via y_e). And maybe ν_2 can turn into μ_R at the other goldstone vertex: (should worry about signs of goldstone couplings, since we sum to the WW diagram)

Maybe we can forget this diagram, since it is parametrically not bigger than the WW diagram, and maybe smaller...

4.4.6 adding up the estimates

We found contributions to the $\mu \to e\gamma$ matrix element of order

$$\sigma_X \sim \frac{esc|\lambda|^2 m_\mu}{16\pi^2 M^2} \tag{4.66}$$

from the exchange of the heavy and light mass eigenstate neutrinos.

1. one could worry that the contributions sum to zero. In the next two sections, we learn from the literature that this is not the case.
2. this contribution can be compared to the light-neutrino-mass contribution, of order
$$\sigma_X \sim \frac{esc|\lambda|^4 m_\mu}{16\pi^2 g^2 M^2}$$
3. the $\mu \to e\gamma$ branching ratio, due to amplitude estimated in (**??**), is
$$\frac{m_\mu^3(\sigma_L^2+\sigma_R^2)/(16\pi)}{G_F^2 m_\mu^5/(192\pi^3)} = \frac{(\sigma_L^2+\sigma_R^2)96\pi^2 v^4}{m_\mu^2} \sim \frac{e^2}{4\pi}\frac{|\lambda|^4 v^4}{\pi M^4} \sim \alpha_{em}\frac{m_\nu^2}{M^2}$$
The current experimental limit is $\lesssim 6\times 10^{-13}$. To reach that rate in the type 1 seesaw, would need $M \sim 10^5 m_\nu \sim$ 10 keV. Maybe our calculation and the type 1 seesaw are doubtful with such small singlet masses....

4.4.7 Conventions of Lavoura paper

Lavoura, hep-ph/0302221, EPJC29(2003)191

Lavoura writes the matrix element

$$\mathcal{M} = \varepsilon^{*\mu}\overline{u}_e(\sigma_L\Sigma_L^\mu + \sigma_R\Sigma_R^\mu + \delta_L\Delta_L^\mu + \delta_R\Delta_R^\mu)u_\mu \tag{4.67}$$

where the lower case $\sigma_L, \sigma_R, \delta_L$, and δ_R are numerical coefficients of mass dimension $1/m$, and

$$\begin{aligned}\Sigma^\nu_{L,R} &= (p^\nu_\mu + p^\nu_e)P_{L,R} - \gamma^\nu(m_e P_{L,R} + m_\mu P_{R,L}) \\ &= i\sigma^{\nu\rho}q_\rho(\sigma_L P_L + \sigma_R P_R)\end{aligned} \tag{4.68}$$

(notice the coefficient of the dipole is the coeff. of $\varepsilon \cdot p_\mu$, provided $\mathcal{M}$ is expanded on operators made of ≤ 1 γs)

$$\Delta^\nu_{L,R} = q^\nu P_{L,R} + \frac{q^2}{m_e^2 - m_\mu^2}\gamma^\nu(m_e P_{L,R} + m_\mu P_{R,L})$$

with $\sigma^{\nu\rho}$ defn of these notes, P+S and C+L. The Δs are irrelevant for on-shell γ pcq $q^2 = 0$ et $q \cdot \epsilon = 0$.

In this notation, have

$$\Gamma(\mu \to e\gamma) = \frac{m_\mu^3(|\sigma_L|^2 + |\sigma_R|^2)}{16\pi} \tag{4.69}$$

Lavoura allows for a gauge interaction of the form

$$\mathcal{L}_{gauge} = W_\alpha \overline{F}\gamma^\alpha(a'_{Li}P_L + a'_{Ri}P_R)f_i + W^*_\alpha\overline{f_i}\gamma^\alpha(a'^*_{Li}P_L + a'^*_{Ri}P_R)F \tag{4.70}$$

(donc $Q_W = +1$ pcq ingoing with f_i) whose coefficients can be obtained by comparing to eqn (4.54). Lavoura takes goldstone ϕ with Lagrangian

$$\mathcal{L}_{goldstone} = \frac{i}{m_W}\phi\overline{F}[(a'_{Ri}m_i - a'_{Li}m_F)P_L + (a'_{Li}m_i - a'_{Ri}m_F)P_R]f_i + h.c. \tag{4.71}$$

I think goldstones are Higgs, despite the i. Lavoura supposes a $W^+_\alpha(p_+)W^-_\beta(p_-)A_\sigma(-q)$ interacton with ingoing momenta

$$ieQ_W[g^{\alpha\beta}(p_+ - p_-)^\sigma - g^{\sigma\alpha}(p_+ + q)^\beta + g^{\sigma\beta}(p_- + q)^\alpha] \tag{4.72}$$

4.4.8 (exact) results (with relative sign) from Lavoura

We need the contributions from the exchange of the heavy neutrino n:

$$\sigma_R = -\frac{1}{3}\frac{ie|\lambda|^2 scm_\mu}{16\pi^2 M^2} \qquad n \text{ exchange} \tag{4.73}$$

and of the light neutrino ν_2

$$\sigma_R = \frac{ieg^2 scm_\mu}{2\cdot 16\pi^2 m_W^2}\frac{-10}{12}\left(-1+\frac{1}{1+\epsilon^2}\right) \simeq \frac{5}{6}\frac{ie|\lambda|^2 scm_\mu}{16\pi^2 M^2} \qquad \nu_i \text{ exchange} \tag{4.74}$$

These are of the same order as our estimates, and do not add to zero.

4.5 from a low E perspective : where did those extra contributions comme from?

We have seen that at energies much below the mass of a particle (leptoquark, W, N...) its effects can be approximated as contact interactions among the propagating particles of your theory. So can we remove the heavy n from our theory, and replace its effects by contact interactions?

Clearly, n contributes at one loop to the dipole operator. When we removed the W from the SM with Dirac neutrino masses, we kept a dipole operator that the W induced at one loop, so surely we can do it for the n too? So in our theory-without-n, we should include, from eqn (4.72)

$$-\frac{1}{3}\frac{ie|\lambda|^2 scm_\mu}{16\pi^2 M^2}\overline{\ell}_e H^* \sigma_{\alpha\beta} P_R \mu F^{\alpha\beta} + h.c.$$

Then there is also the "non-unitary" contribution of ν_m. We can mimic this effect by *augmenting* the ν_m *inverse* propagator by a factor ϵ^2 (that is, we generate a propagator with coefficient $1/(1+\epsilon^2)$ as in eqn (4.73). This can be done by adding the following dimension six operator

$$i\frac{|\lambda|^2}{M^2}\overline{\ell}_m H^{c*} \not{D} H^c \ell_m \tag{4.75}$$

where the coefficient was fixed from eqn (4.73), and there is an i because $i\not{\partial}$ appears in $\mathcal{L}$.

Then of course, there is also the dimension 5 majorana mass operator, which we already introduced. Since the dipole operator is of dimension six, it should be no surprise that we did not get the correct $\mu \to e\gamma$ rate by removing n from the theory and replacing it only with a dimension 5 operator. We see that if we also include some dimension 6 operators in the thoery-without-n, we can get the right answer.

Maybe this all appears ad hoc: we can reproduce the answer when we know what it is. In the next chapter we will see the recipe for consistently removing heavy particles and replacing them with local contact interactions.