5 Continuum Effective Field Theory à la Georgi

- there is a notion that you can describe the physics at any scale by a reasonably simple theory in terms of relevant variables — EFT is a quantitative way to relate the different theories at different scales
- Continuum EFT allows to remove heavy particles and replace them with a tower of local interactions — no nasty non-local form factors!
- Continuum EFT is a technically consistent recipe to "match out" heavy particles in your favourite regularisation scheme = dim. reg. ...
- ... and an intuitively consistent picture of how to put mass scales into "massless" renormalisation schemes like $\overline{MS}$ How many flavours do you put in the QCD β function — and why?
- the idea is that at the mass scale m of a particle η, you remove it from your theory. Then you require that the two theories , with and without η, give "the same" greens functions at the scale m.
  "The same" means equal to whatever order in loops or couplings you compute — in particular, you do your dim reg loop integrals in both theories for momenta to ∞, with the scale $\mu = m$. (so no worries about whether you were supposed to put η back in the loops of the low energy theory for loop momenta $> m$: the answer is no.)
- *read the article by Georgi in Ann.Rev.Nucl.Part.Sci. 43 (1993) 209-252*
  (its infinitely better than anything I say)

Why bother? Why not compute everything in a model?

- sometimes its simpler to *not* deal with a zoo of heavy particles
- EFT is more general — allows to include many models by varying the parameters of the EFT, and perhaps to distinguish the predictions of different classes of models
- if you compute in dim reg, there is a problem with mass scales in the RG logs: they are not there (eg, flavours in the QCD β function). Even if you compute in a model, you need the EFT matching rules to get the RG logs right.

What this chapter aims to do

1. the beautiful intuition of renormalisation with a cutoff à la Wilson
2. but we calculate with dim reg. And dim reg has no physical scale—can we put by hand the correct behaviour at mass threshholds?
   (a) getting the contact interactions right
   (b) log running: which particles contribute to the RGEs?
   (running logs are important for QCD and quark flavour)
3. the matching recipe
   (a) at tree
   (b) at one loop
4. apply the matching recipe to the seesaw
5. RG running of operators: anomalous dimensions, large logs, and photon-mediated μ-e conversion vs $\mu \to e\gamma$.

5.1 Renormalisation à la Wilson

Peskin and Schroeder, chap 12

At first meeting, regularisation and renormalisation are confusing twins who aim to hid infinities so as to get finite results. Miraculously, for several theories, which are said to be renormalisable, this recipe *works*. LEP measured par-per-mil loop corrections in the SM, despite that the loop calculations diverge.

1. tis strange: why does the path integral not depend on short distance fluctuations?

2. why, after renormalisation, does one end up with a curious dependence of the couplings on the energy scale?

Wilson explains:

1. irrespective of the short distance behaviour of the theory, at larger distances, the theory will look renormalisable

2. renormalisation describes the travels of the couplings in parameter space

Why is that?
In chapter 12 of P+S, there is a caln of the path integral in scalar field theory with a cutoff regularisation. There you can see how the parameters of the theory change as the cutoff changes. The cutoff is a physical scale in the theory — and its intuitive that the theory changes with scale. For instance, things look different with a microscope; and you use different theories at the energy scales of the atom and LHC collisions.

1) When you regularise with a cutoff, and the cutoff passes below the mass of a particle in your theory, then the particle can no longer propagate on-shell (since its momentum is limited by the cutoff), and it can be removed and replaced by contact interactions. So this explains why most theories appear renormalisable — non-renormalisable interactions are suppressed by a higher mass scale, which makes them more and more **irrelevant** (in Wilsons classification) as the cutoff decreases. Whereas renormalisable interactions, stay **marginal**.

2) Short distance fluctuations do matter (a little); the parameters of your theory depend on the scale of fluctuations which are included in them.

5.2 But we calculate in dim reg

Dimensional regularisation has a mass scale μ which appears to keep the mass dimensions right. Despite that it appears in logs, it is not a "physical scale" in the sense that dim reg never compares it to masses in the theory.

For instance, when you compute the one-loop corrections to the gluon propagator (these contribute to the β-function of QCD, which tells how the strong coupling evolves with energy scale), you include a quark bubble. The mass of the quarks in the bubble is irrelevant for the $1/\varepsilon$ divergence, and its associated $\ln(4\pi\mu^2)$ which contributes to the β-function. So dim reg *does not know*, when running the strong coupling at energies of 5 GeV, whether or not to include the top quark. You have to tell it.

You have to tell dim reg, for each range of energies, what are the available degrees of freedom. That is, dim reg operates as if every particle in the theory was massless. Thats fine, its a very useful regularisation scheme, so we live with its idiosyncrasies and only give it particles which can be treated as massless.

So you can imagine the following:

1. start at some high scale with a fundamental theory,
2. use your RG equations to run the parameters of the theory down in energy... until you meet the mass of the heaviest particle in the theory (in our seesaw, that was n).
3. at M_n, you remove n from the theory, replacing it, in the theory below M_n, by contact interactions (up to some dimension you specify as a function of the desired accuracy of your calculation).
4. the coefficients of the contact interactions can be computed (to some order in coupling and loop expansions), by matching (at M_n) the greens functions of the theory from above M_n to the theory below M_n. Notice that loops, in both theory, are always integrated over momenta to ∞.
5. then you use RGs to evolve your new theory down towards the next mass scale (m_W, in our seesaw example). Notice that the coefficients of contact interactions are couplings in the EFT, they can have $1/\epsilon$ divergences that one subtracts, and $\log(4\pi\mu)$s which allow them to "run" in the RGEs (just like renormalisable couplings).

In this way, you get correct coefficients for contact interactions, and correct thresholds in your RG logs.

5.3 The matching formalism(Georgi on-shell EFT)

Georgi, NPB 361(1991) 339

At the scale Λ, have two theories, which get refered to in various ways:

1. the theory above Λ = the full/heavy theory. Contains the light particles λ, and the particles Φ of mass $\simeq \Lambda$, with Lagrangian

$$\mathcal{L}_{>} = \mathcal{L}(\lambda) + \mathcal{L}(\Phi, \lambda)$$

2. the theory below Λ without the heavy particles = the light/effective theory. The Lagrangian is

$$\mathcal{L}_{<} = \mathcal{L}(\lambda) + \delta\mathcal{L}(\lambda)$$

The aim is to determine the Lagrangian of the effective theory (the additions $\delta\mathcal{L}(\lambda)$), which should have the following properties

$$\begin{aligned} &\text{local} \\ &\text{respects all symmetries of the light theory} \\ &\text{reproduce the S} - \text{matrix elements of the heavy theory} \end{aligned} \tag{5.76}$$

so can expand $\delta\mathcal{L}(\lambda)$ on all gauge-etc-invariant contact interactions (often refered to as effective operators) $\{\mathcal{O}^I(\lambda)\}$, constructed out of light fields λ, with arbitrary coefficients C^I. This zoo can be organised in inverse powers of Λ:

$$\delta\mathcal{L}(\lambda) = \sum_{n=1}^{\infty} \frac{C_I^{(4+n)}}{\Lambda^n}\mathcal{O}^I(\lambda) \tag{5.77}$$

usually, one is only interested in the lowest dimension operators, suppressed by 1 to 4 powers of Λ.

At each order in $1/\Lambda$, the set of effective operators satisfying (5.76) can be large. The set can be reduced by using the full equations of motion (see section 5.8):

$$\not{D}\ \ell - y_e H_d^* e_R = 0 \qquad \Rightarrow \qquad \overline{\ell}\sigma_{\rho\eta}\not{D}\ \ell F^{\rho\eta} = y_e \overline{\ell} H_d^* \sigma_{\rho\eta} e_R F^{\rho\eta}$$

5.4 The matching recipe (Georgi on-shell EFT)

Determine the C^I by matching (= equating at the renormalisation scale $\mu = \Lambda$) the 1PI Greens functions involving light fields in the light and heavy theories. Define 1PI in the light theory. Do this matching at desired order in all perturbative expansions (couplings, $1/\Lambda$, loops).

Example 1: leptoquark S_0
The scale Λ corresponds to the S_o mass m_S.
Perturbative expansion in λ of light-leg-diagrams mediated by S_o starts at λ^2
At tree level, S_o induces 4-fermion interactions, as given to the left in eqns (1.17) and (1.18)):

$$-i\lambda_{LS_o}(\overline{(u_L)^c}e_L)\frac{i}{p^2-m_S^2}(+i)\lambda^*_{LS_o}(\overline{(d_L)^c}\nu_L)^\dagger = -i\frac{|\lambda_{LS_o}|^2}{m_S^2}\left[1+\frac{p^2}{m_S^2}+...\right](\overline{(u_L)^c}e_L)(\overline{(d_L)^c}\nu_L)^\dagger = -i\frac{|\lambda_{LS_o}|^2}{m_S^2}(\overline{(u_L)^c}e_L)(\overline{(d_L)^c}\nu_L)^\dagger$$

$$-i\lambda_{RS_o}(\overline{(u_R)^c}e_R)\frac{i}{p^2-m_S^2}(+i)\lambda^*_{LS_o}(\overline{(d_L)^c}\nu_L)^\dagger = -i\frac{\lambda_{RS_o}\lambda^*_{LS_o}}{m_S^2}\left[1+\frac{p^2}{m_S^2}+...\right](\overline{(u_R)^c}e_R)(\overline{(d_L)^c}\nu_L)^\dagger = -i\frac{\lambda_{RS_o}\lambda^*_{LS_o}}{m_S^2}(\overline{(u_R)^c}e_R)(\overline{(d_L)^c}\nu_L)^\dagger$$

We want the effective light theory to be *local* (therefore we do not consider to write a form factor $F(p^2) = \lambda^2/(m^2-p^2)$. Local theories are nice; they are what is in QFT books). So we write the LQ propagator as the middle formula above, then we focus on the lowest order term in the $1/m_S^2$ expansion, and obtain the contact interaction to the right. That's what we guessed as the leading result in chapter 1!

Could go to higher order:
1— in the loop expansion eg, LQ induce $\mu \to e\gamma$ at $\mathcal{O}(\lambda^2 e/(16\pi^2 m_S^2))$
2— in $1/m_S^2$: To obtain effective operators corresponding to the $1/m_S^4$ terms above: judiciously apply derivatives to the fermion fields such as to obtain the —4-p$|^2$ of the LQ, then remember to use EoM of fermions to check what the operator is equivalent to
3— in λ eg, can get 4-q or 4-lepton operators from box diagrams at order $\lambda^4/(16\pi^2 m_S^2)$

NB: not seem to care, in tree-matching, about the scale we match at? No surprise, scale dependence of couplings arises at loop...

Reminder from Ramond— what the 1PI action?

Pragmatically, **1PI means that the diagrams do not fall apart if you cut one line**. In particular, 1PI diagrams are "amputed", that is, have no external legs. Therefore, for example, in the SM, muon decay by W exchange is not 1PI, because you have two W propagators if you cut the W line. But in the effective Fermi theory, muon decay is 1PI, because the diagram is the blob corresponding to G_F.

To define the 1PI action

- In Ramond notation, $W[J]$ is the path integral over the field ϕ, who is coupled to the external current J. J can "probe"/poke the field, to learn how it behaves. W can be expanded in powers of J; this gives all diagrams (might be the S-matrix, assuming one normalised by the vacuun to vacuum transition amplitude?). Since J "pinches" the field, the resulting disturbance has to propagate, so expansions in J have external legs.

  The connected diagrams are generated by $Z[J] = logW[J]$. P+S identify

  $$W[J]_{P+S} = Z[J]_R \qquad E[J]_{P+S} = Z[J]_R$$

  The Effective Action, $\Gamma[\phi_{cl}]$ for Ramond, is the generator of 1PI diagrams, which do not have external legs. It can be expanded in powers of the "classical field", which is the expectation value of ϕ:

  $$\phi_{cl} = \frac{\delta}{\delta J} Z \tag{5.78}$$

  and $\Gamma[\phi_{cl}]$ can be obtained as the Legendre transformation of Z: $\Gamma[\phi_{cl}] = Z[J] - \int J\phi_{cl} d^4x$.

- See P+S 11.63 for how to compute $\Gamma[\phi_{cl}]$ from the *connected* diagrams — which are easy to get by doing a perrturbative-in-a-coupling-constant expansion of the path integral (in practise what Ramond does in chapter IV, despite sections at end of chapter III about ζ-fn, computing ϕ_{cl}, etc). In section 11.5, P+S show that an expansion of Γ in terms of ϕ_{cl} (without computing ϕ_{cl}; they just use chain rule) gives 1PI diagrams, which are sufficient to determine the theory. For this reason, I assume, Georgi in EFT matches at 1PI. Notice, however, that 1PI $\neq$ S-matrix!.

5.5 Matching in the seesaw

In the type 1 seesaw, we wish to remove the heavy singlet n and the W from our effective theory, so we will have to perform matching sequentially at the scales $\Lambda = M_n$, and $\Lambda = m_W$. In addition, the dipole operator arises at one loop, so we will have to match consistently at one loop.

We stay always in the broken EW theory. One might think that at scales $\gg m_W$, one should get the same answer more simply in the unbroken EW theory. However, there are subtleties in the matching recipe, related to the defn of 1PI(the free charged leptons in unbroken SU(2) are chiral, whereas in the broken SM they have Dirac masses. So a Higgs insertion on an external fermion line, which just flips chirality in the broken theory, makes the diagram 1P-*reducible* in the unbroken theory.). Doing a broken EW caln for n, as guesstimated in the previous chapter, will give the right answer. Its just messier.

Caveat: the Effective Lagrangian without n is constructed by doing several perturbative expansions

- expand in e, λ, the charged lepton yukawas y_α, $1/M_n$, and $1/(16\pi^2)$. Lets not try to do perturbation theory in g or v, because they confusingly can appear upstairs and downstairs.
- we know the leading contribution to $\mu \to e\gamma$ is $\mathcal{O}(e\lambda^2 y_\mu/[16\pi^2 M^2])$. So we can neglect operators whose coefficients are of $\mathcal{O}(e^2)$, or $\mathcal{O}(\lambda^{3+..})$, $\mathcal{O}(y^{2+})$, or $\mathcal{O}(1/M^{3+})$.

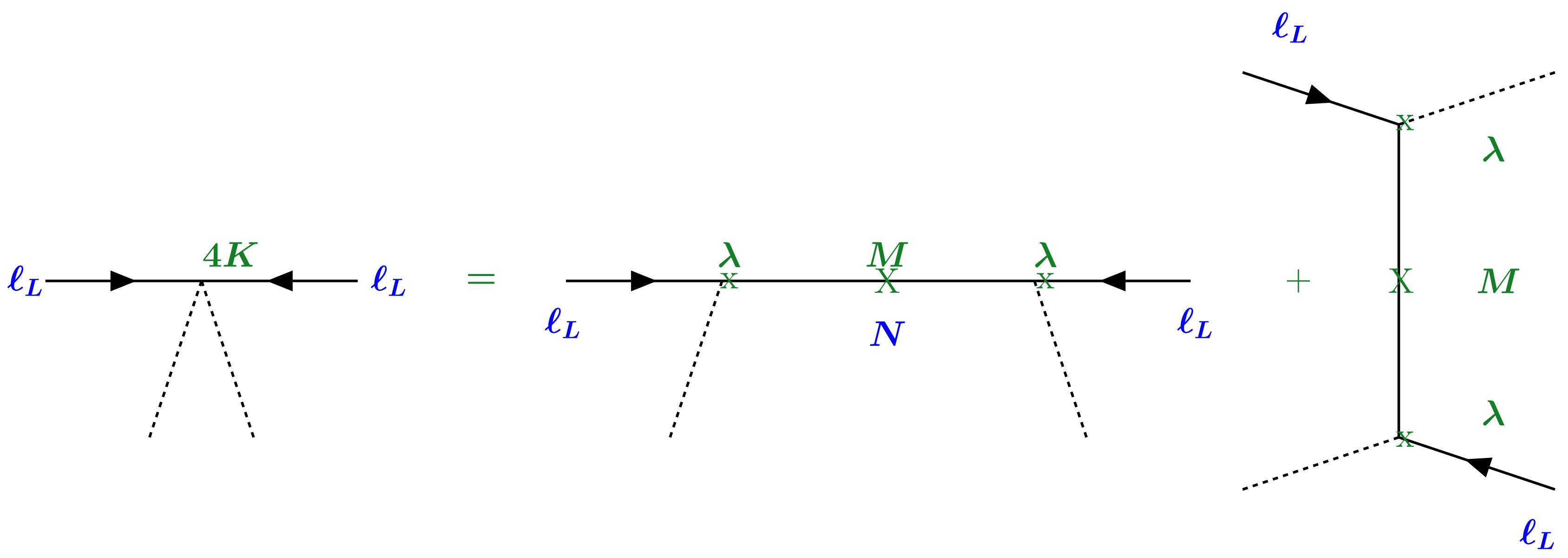
Notice that, *in principle* (but not here at lowest order), perturbative expansions in several parameters require a great deal of care. Sometimes the domininant contribution is higher order in a large parameter, because that allows lower order in small parameters...

Matching out the n

1. at dimension five, there is only one allowed operator (by gauge invar, etc) = the operator which gives majorana masses to the light neutrinos, which can be written in SM gauge invariant form as $\ell_\alpha H_d \ell_\beta H_d$.

The dim five tree level operator

For this tree-level operator, we work in the unbroken EW theory (the difference is higher order in $1/M$, since N and n differ by ϵ). Suppose we write $+\sum_{\alpha\beta}^{3} \frac{K_{\alpha\beta}}{M} \ell_\alpha H_d \ell_\beta H_d$ in the effective Lagrangian; there is a 4 in the F-rule for the two identical Higgs and lepton fields. There are two diagrams, for any flavour combination, in the full theory:



$$i\frac{4K_{\alpha\beta}}{M} = -i\frac{\lambda_\alpha\lambda_\beta}{M} - i\frac{\lambda_\alpha\lambda_\beta}{M}$$

$$\Rightarrow K_{\alpha\beta} = -\frac{\lambda_\alpha\lambda_\beta}{2}$$

Note, in passing, that the 2s work for the light neutrino mass term in the Lagrangian

$$\frac{K_{\alpha\beta}}{M}\langle H_o\rangle^2 \nu_\alpha\nu_\beta = \frac{\lambda_\alpha\lambda_\beta\langle H_o\rangle^2}{2M}\nu_\alpha\nu_\beta \leftrightarrow \frac{1}{2}\frac{m_D^2}{M}\nu\nu$$

We keep this operator; it is or $\mathcal{O}(\lambda^2)$, and $\mathcal{O}(1/M)$.

2. at dimension six, there are many possible operators.

- At tree-level, the $N - \nu$ mixing reduces the normalisation of the light field ν_m. We already saw that this can be described by the "non-unitary operator":

$$i\frac{|\lambda|^2}{M^2}\overline{\ell}_m H_d^{c*} \not{D} \; H_d^c \ell_m$$

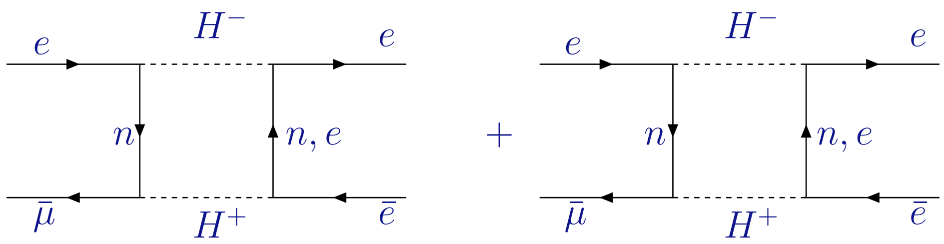
where the coefficient was fixed from eqn (4.74).

- at one loop, can draw Feynman diagrams for many more effective interactions. But we are only interested in the dipole! Because anything else will contribute to $\mu \to e\gamma$ at two-loop...There is caveat about RG running; more later. The matching rules say that in the full and effective theory, should compute all the 1 loop diagrams, with all the dynamical particles and available interactions (including the tree contact interactions obtained already). Since we include the "non-unitary" operator in the effective computation of the light neutrino loops, these light loops will give the same contribution above and below M. So we only need the dipole coefficient from n exchange that we already obtained in section 4.5 :

$$\sigma_R = -\frac{1}{3}\frac{ie|\lambda|^2 scm_\mu}{16\pi^2 M^2} \quad \Rightarrow \quad -\frac{1}{6}\frac{ie|\lambda|^2 scm_\mu}{16\pi^2 M^2}\overline{\ell}_e H_d^* \sigma_{\alpha\beta} P_R \mu F^{\alpha\beta} + h.c.$$

(since coeff is $\sigma_X/2$ because $F \sim 2qA$).

- at one-loop, there are also various boxes, contributing for instance to $\mu \to e\bar{e}e$:



which we can neglect because they are higher order in Yukawas. And could only contribute to $\mu \to e\gamma$ via another loop.

Matching out the W

In principle, when we match out the W, it produces the Fermi interaction, and various LFV decays such as $\mu \to e\bar{e}e$. However, we are only interested in $\mu \to e\gamma$, so we only consider (additional) contributions to the coefficient of the dipole operator (because there is already the heavy neutrino contribution).

We know the answer from Lavoura. There are the $W+$ goldstone diagrams for the two light neutrinos, and the leading order contributions don't quite cancel (so the GIM mechanism does not quite work for majorana masses) because the non-unitary operator reduces the normalisation of ν_m. So below the scale m_W, the additional contribution to the coefficient of the dipole operator is

$$\sigma_R = \frac{ieg^2 scm_\mu}{2 \cdot 16\pi^2 m_W^2} \frac{-10}{12} \left(-1 + \frac{1}{1+\epsilon^2} \right) \simeq \frac{5}{6} \frac{ie|\lambda|^2 scm_\mu}{16\pi^2 M^2} \qquad (light) \tag{5.79}$$

and the sum of the heavy and light contributions is

$$\sigma_R = \frac{1}{2} \frac{ie|\lambda|^2 scm_\mu}{16\pi^2 M^2} \qquad (n + light\ \nu) \tag{5.80}$$

Notice that there is only a contribution for singlet μ_R decay, because the weak interactions involve the doublets, so the chirality flip is via mass insertions on the external legs, and m_e is negligeably small.

5.6 Matching out the W at one loop in QCD

Buras at Les Houches 9806471
eqns 5.23 - 5.28

Here consider an example not related to this course, but relevant to quark flavour physics. The purpose is to see what to do with divergences that arise due to the presence of a non-renormalisable interaction in the EFT.*numerical factors tbc!*

Consider a 4-quark interaction $\overline{c}b\overline{s}c$, mediated at tree level by the W. The "full" theory is the SM, the effective theory is QED*QCD, with a 4-quark contact interaction.

1. at tree level, can "match out" the W, as we did for the leptoquark, to get a contact interaction

$$Q_2 \equiv \frac{4G_F}{\sqrt{2}}(\overline{c}^\alpha \gamma^\mu P_L b_\alpha)(\overline{s}^\beta \gamma^\mu P_L c_\beta)$$

   where α, β are colour indices

2. now suppose that you wish to do a more accurate calculation, and match at one-loop in QCD. So you can decorate the contact interaction in the EFT with gluon exchange, and do the same for the W-exchange diagram in the SM.

   - you can forget about the diagrams with a gluon emitted and absorbed on the same leg, because they are the same in the full and EFT.
   - you discover that you can exchange colour between the two fermion lines, so you will generate a new contact interaction

$$Q_1 \equiv \frac{4G_F}{\sqrt{2}}(\overline{c}^\beta \gamma^\mu P_L b_\alpha)(\overline{s}^\alpha \gamma^\mu P_L c_\beta)$$

   - there are $1/\epsilon$ divergences in the full theory (because we dropped the wave fn renorm diagrams), which are identical for the EFT
   - there are $1/\epsilon$ divergences that only appear in the EFT.

But the divergence is ok! The operator coefficients are unknown parameters, so put a counter term that subtracts the $1/\epsilon$ (just like for renorm couplings), and match the finite part to the full theory.

With the $1/\epsilon$, there is always a log — the coefficient of the log is the anomalous dimension ($\sim$ "β_0") of the operator coefficient, it tells you how the coefficient runs with scale (eg $m_W \to m_b$).

Not only do operator coefficients "run", like couplings, they can also "mix" into each other...as this example shows: the QCD loops transforms $Q_2 \to Q_1$. So all the gluon loops between $m_W \to m_b$ will "mix" the operators, so the RGEs are a big matrix...

Summary: in dim reg, the divergences are of the form $1/\epsilon + \log$; the $1/\epsilon$ can be subtracted, the finite part of the coefficient is fixed by matching, and the log gives RG running. This is true for renormalisable, or not, couplings. So the EFT is just as well behaved as a renormalisable theory.

The great advantage of renormalisable theories, is that there are a finite number of terms in the Lagrangian. Once you have fixed their coefficients by matching to observables, you ca predict everything. In an EFT, there are an infinite tower of terms, so you can fit everything, and predict nothing.

5.7 Operator bases: Buchmuller-Wyler (with polish update)

Buchmuller+Wyler NPB268[1986) 621

Grzadkowski etal 1008.4884

There is a list of SM gauge invariant operators of dimension five and six, in Buchmuller Wyler. They use the full Eqns of Motion to simplify the operator basis. For instance, from a Lagrangien containing

$$\mathcal{L} = \overline{\ell}(i\not{D}\ \ell - y_e H e_R) - y_e^* \overline{e_R} H^\dagger \ell \tag{5.81}$$

$$= -i(D_\alpha \overline{\ell})\gamma^\alpha \ell - y_e \overline{\ell} H e_R - y_e^* \overline{e_R} H^\dagger \ell \tag{5.82}$$

BW obtain EoM

$$i\not{D}\ \ell - y_e H e_R = 0 \qquad - i(D_\alpha \overline{\ell})\gamma^\alpha - y_e^* \overline{e_R} H^\dagger = 0 \tag{5.83}$$

Notice this differs from using the free eqns of motion that apply to external legs!

The polish list (slightly shorter than BW), contains 15 + 19 + 25 = 59 baryon-number-conserving operators, counted with 0, 2 or 4 fermion fields. The different flavour choices and contractions are in addition.

5.8 Using the Eqns of motion to reduce the basis

Simma, ZPC61(1994)67 [hep-ph/9307274

See figure 1 of the paper of Simma. It explains diagrammatically, why you can use the EoM to reduce the operator basis. The point is that the S-matrix element of an operator containing the EoM vanishes.

For example, imagine an operator of dimension 6

$$(\overline{\not{D}\ \ell})H_d(\not{D}\ e) \tag{5.84}$$

in a massless theory (ie, neglect Yukawas for simplicity). Focus on the $\not{D}\ \ell$ part. The Eqns of motion are

$$i\not{D}\ \ell = i\gamma^\mu(\partial_\mu - ig\vec{\tau}\cdot\vec{W}_\mu)\ell = 0$$

The operator of eqn (6.93) can participate in various diagrams contributing to different S-matrix elements:

- consider $\langle e\bar{\mu}|H\rangle$, the operator contributes, the $\not{D}\ \ell$ gives $\not{p}\ e_L$ where the e_L is an external leg. Since the external legs satisfy the free EoM $\not{p}\ u = 0$, these diagrams vanish.

- consider $\langle e\bar{\mu}|WH\rangle$, there is a a diagram with our operator and a W on the external leg (Recall that are drawing Feynman diagrams for S-matrix, so not limited to 1PI!). $\not{D}\ \ell$ gives $\not{p}\ \nu_L$, but now the internal ν_L has a propagator, which is cancelled by the $\not{p}$ from the operator, and at $\mathcal{O}(g)$, its other vertex is $W\bar{e}\nu$, which comes from $\vec{\tau}\cdot\vec{W}_\mu)\ell$. So this diagram will contribute to the same S-matrix element, at the same order in g, as the $WHe\mu$ contact interaction obtained from $\not{D}\ \ell \to \gamma^\mu W_\mu \ell$.

  The claim of Simma, is that these two amplitudes have opposite sign, so cancel.

So the point seems to be that, even if NP gives an on-shell operator like eqn (6.93), that is, a Feynman rule where gauge bosons are emitted, there is no contribution to the S-matrix, because the contact contribution is cancelled by W emission from an external leg.

Selon Simma, who does an example for bsg penguins, can match unambiguously to a complete set of operators, by matching off-shell Greens functions (cad, can tell which diagrams contribute to which operators, despite same external legs). Then remove operators who differ by EoM, and on-shell coefficients are unambiguous. The choice of a basis of operators is *not* unambiguous, but theories with different choices of operator basis will be "on-shell equivalent", cad give same S-matrix, provided that the basis operators differ only by operators that vanish by EoM (or are BRS transforms of some other operator).

5.9 For instance, the non-unitarity operator is not in the BW basis

The "non-unitarity operator", which we used, can be transformed into Z and W penguin operators :

$$\begin{aligned} i\overline{H^c\ell}\partial\!\!\!/ \, (H^c\ell) &= i\overline{H^c\ell}\partial\!\!\!/ \, (H^c\ell) \\ &= i\overline{H^c\ell}[i(D\!\!\!\!/ \;\; H^c)\ell + H^c iD\!\!\!\!/ \;\; \ell] \\ &= \overline{H^c\ell}[i(D\!\!\!\!/ \;\; H^c)\ell + \mathcal{O}(y_e)] \\ &= i\overline{H^c\ell}(D\!\!\!\!/ \;\; H^c)\ell \end{aligned} \tag{5.85}$$

On peut utiliser maintenant une identité SU(N)

$$t^a_{\alpha\beta}t^a_{\delta\epsilon} = \frac{1}{2}(\delta_{\alpha\epsilon}\delta_{\delta\beta} - \frac{1}{N}\delta_{\alpha\beta}\delta_{\delta\epsilon}) \tag{5.86}$$

réecrit comme

$$\delta_{\alpha\epsilon}\delta_{\delta\beta} = \frac{1}{2}\left(\tau^a_{\alpha\beta}\tau^a_{\delta\epsilon} + \delta_{\alpha\beta}\delta_{\delta\epsilon}\right)$$

to get something like (signs not checked)

$$= \frac{i}{2}\left[(\overline{\ell}\vec{\tau}\gamma_\alpha\ell)((D_\alpha H^c)\tau H^{c*}) + \overline{\ell}\gamma_\alpha\ell((D_\alpha H^c)H^{c*})\right] \tag{5.87}$$

which, using $((D_\alpha H_u)H_u^*) = (H_u^\dagger D_\alpha H_u)$, look like thr Z and W penguin operators given in the BW list involving vectors, fermions and scalaires :

$$\begin{aligned} O^{(1)}_{H\ell} &= i(H^\dagger D_\alpha H)(\overline{\ell}\gamma^\alpha\ell) && (5.88) \\ O^{(3)}_{H\ell} &= (H^\dagger D_\alpha\tau_A H)(\overline{\ell}\gamma^\alpha\tau_A\ell) && (5.89) \end{aligned}$$

These are called W and Z penguins, because if the Higgs takes its vev, these are fermion-fermion-gaug-boson vertices.